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Review
On the Twyman effect and some of its applications

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This review is a survey of a number of papers on the Twyman effect, discovered in 1905, in the processing of optical glass. The subject of these works is a detailed study of the Twyman effect in optical glass, quartz and sapphire crystals, and metals in the form of thin wafers with a thickness of 1 to 0.1 mm. It has been established that plane-parallel wafers of the studied materials, ground on one side and polished on the other, bend until the surface becomes strictly spherical or elliptical, concave on the polished side with a radius of curvature in a quadratic dependence of the wafers' thickness and not depending on their size and shape. A phenomenological conclusion of the main law has been drawn. It reveals the physical sense of the Twyman effect through its connection with the surface tension of solids. Three new methods have been worked out which are rather interesting prospective applications of the Twyman effect (for studying anisotropy in crystals, for processing concave spherical mirrors, for measuring apparatus and for preparing a new type of spherical crystalline diffraction lattice for X-ray spectroscopy).

1. Introduction
The Twyman effect is an interesting phenomenon, first observed by the English optician, Twyman, in 1905. The phenomenon can be stated simply as follows: when a plane-parallel wafer cut out of optical glass with a thickness of several millimetres is polished on one side and ground on the other, it bends, the polished side becoming concave.

A complete investigation of the Twyman effect was not undertaken until 10 years ago. Although some of its aspects have been studied in a small number of works [1–6], devoted to this phenomenon in optical glass, as well as in some semiconductors, a plausible physical explanation of the causes of this effect has not been given. It is supposed by most authors that the phenomenon is due to the microfissures in the ground side, created by the abrasive powder. Twyman himself has not investigated this phenomenon further. Like other authors, he considered it a difficulty in optical industry in the production of some optical details.

This paper is a review of a number of our works [7–13, 15, 17], whose object is a detailed study of the Twyman effect in optical glasses, quartz glass, quartz and sapphire crystals and metals. The strict physical law to which this effect is subjected has been experimentally determined. A new, more plausible explanation of the causes of the Twyman effect has been put forward. A phenomenological conclusion has been made, confirming the experimentally obtained law of the Twyman effect. In addition, the relation of this effect with the surface tension in solids has been shown.

Some interesting applications of the Twyman effect have been given.

2. Experimental studies on the Twyman effect in glasses, crystals and metals
Using the technology and processing suggested by us [7], it was possible to obtain even the thinnest plane-parallel wafers of an order of 0.1 mm, even 0.05 mm, cut out of optical glass and crystals, polished on one side and ground on the other, without the Twyman effect manifesting itself until the end of the processing. This processing tech-

TABLE I Dependence of the radius of curvature, R, on the diameter of wafers, $2r$, with different thicknesses

$h = 0.5$ mm		$h = 0.3$ mm	
$2r$ (mm)	R (m)	$2r$ (mm)	R (m)
15	19.80	15	5.78
12	19.65	12	5.65
8	20.41	8	5.78

TABLE II Values of the material constant, a, from law $R = ah^2$ of the Twyman effect

Material	Quartz glass	BK_7	BaK_2	SF_4
a (10^{-7} m^{-1})	7.0	5.8	5.3	4.4

nology gave us the opportunity of making systematic studies of the Twyman effect that enabled us to get closer to its physical nature.

The final experimental studies were conducted by measuring the diameters of the Newton rings, obtained during the interference in monochromatic light, reflected by the bent concave surface of the wafer and a plane or a concave optical glass standard.

The Twyman effect was studied above all with wafers cut from seven to eight kinds of glass – several types of optical glass and quartz glass. It was experimentally determined that there is a strict repetition of the observed phenomenon of spherical bending of wafers of different form, size and thickness. A series of samples consisting of a great number of wafers in a round, square or rectangular shape of different size and thickness varying between 0.1 and 1 mm was prepared for this purpose. The wafers were produced with great precision, with differences in the thickness at different points on the wafers' surface up to 1 μm.

Experiments showed that the radius of the spherical surface of the wafers does not depend either on their shape or their size, but solely on their thickness and the characteristics of the material.

Table I gives the radii of curvature, R, for round wafers of quartz glass with a thickness $h =$ 0.5 and 0.3 mm and different diameters, $2r$. It can be observed that with the same thickness, the radius of the spherical surface does not depend on the wafer's diameter.

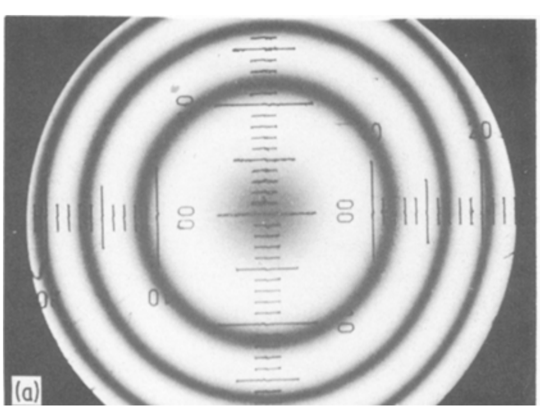
Figs. 1a and b are photographs of the Newton interference rings for wafers of quartz glass with a thickness $h = 0.2$ and 0.1 mm, respectively. It can be seen that with the smaller thickness the bending is noticeably greater: the Newton rings are greater in number and density.

Fig. 2 shows graphically, on a double log scale, the dependence of the wafers' radius of curvature on their thickness, for several kinds of optical glass and (for comparison) for quartz and sapphire monocrystal cut out perpendicularly to Z. As can be seen in the figure the dependencies, $R(h)$, are strictly linear on a log scale, the slope of the straight lines being the same for all optical glasses and monocrystals that have been studied.

It was determined from these experimental lines that the dependence $R(h)$ for all optical glasses studied is a square parabola:

$$R = ah^2, \tag{1}$$

where a is a new typical material constant of the substance, depending on Young's modulus, E, the Poisson coefficient, μ, and the surface state. Its values are given in Table II.



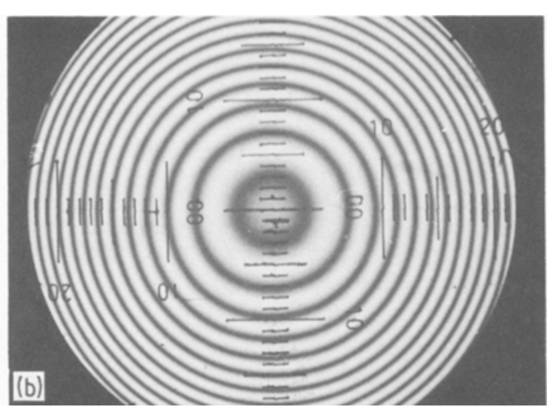


Figure 1 Photographs of the interference rings for wafers of quartz glass of thickness $h = 0.2$ mm (a) and $h = 0.1$ mm (b).

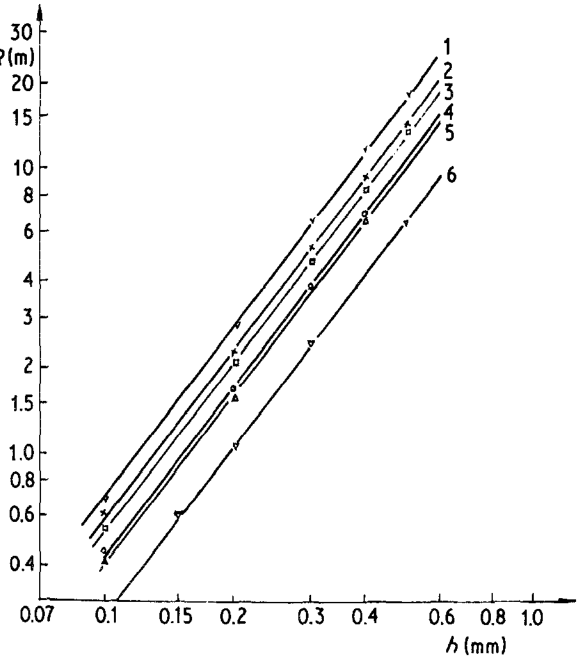


Figure 2 Dependence of the radius of curvature on the wafers' thickness: 1, quartz glass; 2, BK_7; 3, BaK_2; 4, SF_4; 5, quartz monocrystal with face (0 0 0 1); 6, sapphire monocrystal with face (0 0 0 1).

It was established that wafers cut out of quartz and sapphire monocrystals also bend according to the same law. The bending of a wafer, cut out perpendicular to the Z-axis is strictly spherical, while that of wafers cut out parallel to Z is elliptical. Moreover, for wafers perpendicular to X, the large axis of the ellipse, is parallel to Z. Fig. 3 shows a plot on a double log scale, of the dependencies $R(h)$ for a wafer of a quartz monocrystal, cut out perpendicular to Z, as well as $R'(h)$ and $R''(h)$ for a wafer, cut out perpendicular to X, where R' and R'' are the radii of a curvature in the direction of the large and the small axes of the ellipse, respectively.

Figs. 4 and 5 are photographs of interference rings for two quartz wafers, cut out perpendicular to the Z-axis and for two others, cut out perpendicular to the X-axis at two different thicknesses. The rings, circular and elliptical, are compared for equally thick wafers: $h = 0.2$ mm (Fig. 4) and $h = 0.1$ mm (Fig. 5). As seen, with thinner wafers the interference rings are greater in number and density, since bending is greater in accordance with the law given in Equation 1.

The Twyman effect was also studied on different metals [11]. It has been established that wafers of copper and silver steel bend in a strictly spherical surface with the polished side concave as a result of the treatment by the above technology [7]. The dependence of the wafers' radius of curvature on their thickness is shown graphically, on a double-log scale, in Fig. 6 (1, for copper wafers; 2, for silver steel wafers). As can be seen in the figure, the dependencies $R(h)$ are strictly linear on a log scale, the slope of the straight lines being the same for both metals investigated and coinciding

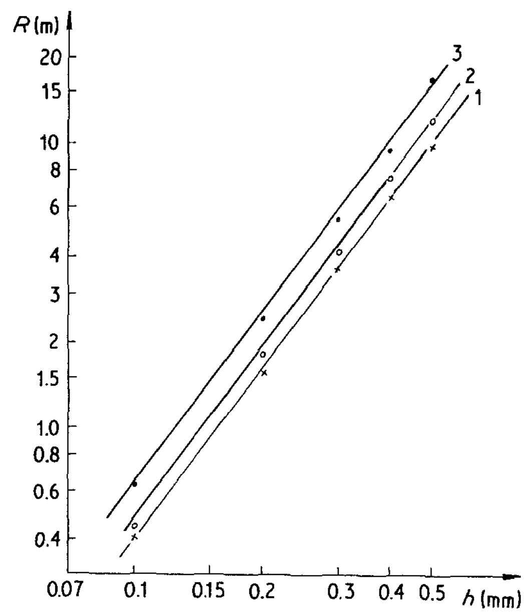


Figure 3 Dependence of the radius of curvature on wafer thickness: line 1, $R(h)$ for a wafer of quartz crystal cut out perpendicular to Z; lines 2 and 3, $R''(h)$, and $R'(h)$ for wafers perpendicular to X along the small and the large axes, respectively.

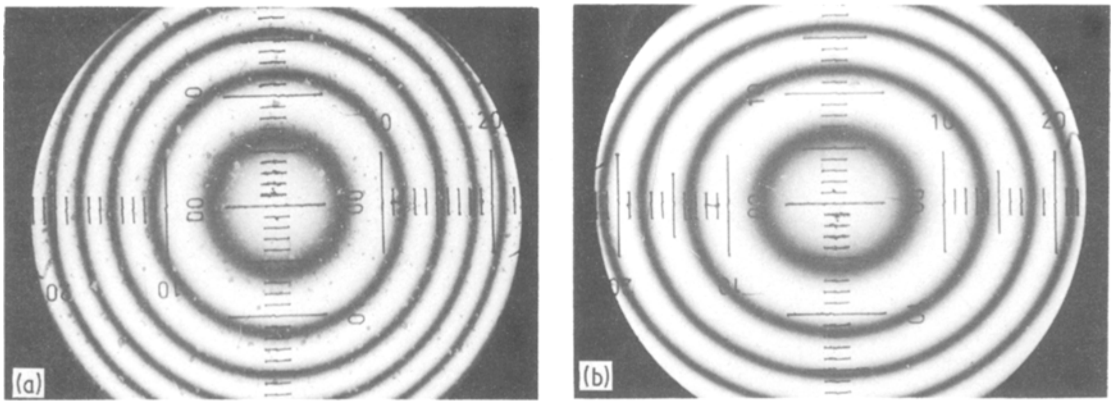


Figure 4 Photographs of the Newton rings obtained from wafers of a quartz crystal of thickness $h = 0.2$ mm, cut out: (a) perpendicular to Z, (b) perpendicular to X.

with that for optical glasses and crystals (Figs. 2 and 3). It follows therefrom that the law (Equation 1) also remains in force for metals. It should be noted, however, that the values of R for the same wafer thickness in metals are smaller by one order of magnitude than in optical glasses and crystals (from 0.04 to 2.00 m). This shows that the Twyman effect is much more pronounced in metals. This is probably due to the greater intermolecular forces of interaction on the wafers' polished side.

The influence of temperature on the Twyman effect was also investigated [12]. Quartz monocrystal wafers with (0 0 0 1) planes were investigated in an annealing regime at a heating velocity of 6° C min^{-1}, annealing temperature 500° C (heating for 30 min) and a cooling velocity of 4° C min^{-1}. The results showed that the radius of curvature changes insignificantly when heated under these conditions, the law (Equation 1) of the dependence of R on wafer thickness remaining in force, as can be seen from Fig. 7. Analogous results were obtained with the annealing of optical glass SF_{14} for 30 min at 300° C, heating velocity 6° C min^{-1} and cooling rate 2° C min^{-1}, as shown in Fig. 8. This fact tends to confirm our assumption that the Twyman effect is caused mainly by the surface tension of the wafers' polished side and not only by the inner stresses, produced by different treatment of the two sides of the wafer, which may be expected to disappear after annealing.

3. Theoretical conclusions. The Twyman effect and its relation to surface tension in solids

The experimentally determined facts show that the bending of the wafers is exactly spherical and the radius of curvature, R, depending on the

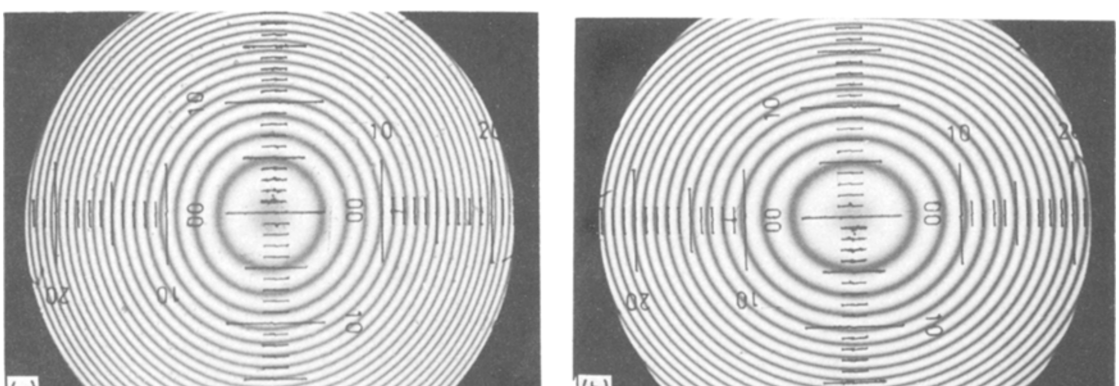


Figure 5 Photographs of the Newton rings obtained from wafers of a quartz crystal of thickness $h = 0.1$ mm, cut out (a) perpendicular to Z, (b) perpendicular to X.

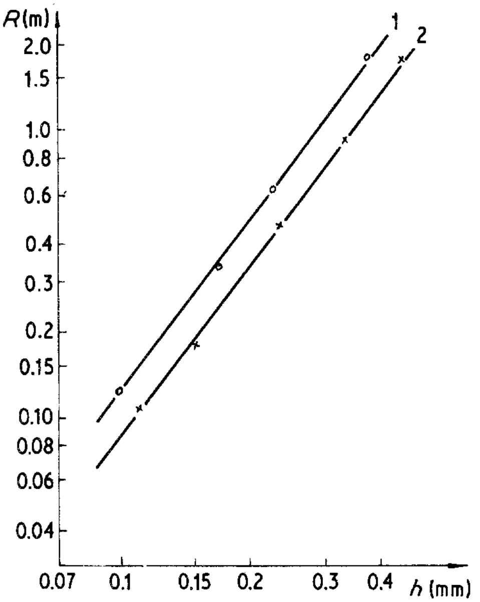


Figure 6 Dependence of the radius of curvature on the thickness of the metal wafers (line 1 for copper wafers; line 2 for silver steel wafers).

wafers' thickness, changes according to a strictly determined law (Equation 1). These facts, as well as a number of additional investigations of this phenomenon, give us grounds to suppose that the bending is due to the unilateral surface tension on the wafer's polished face, which is no longer

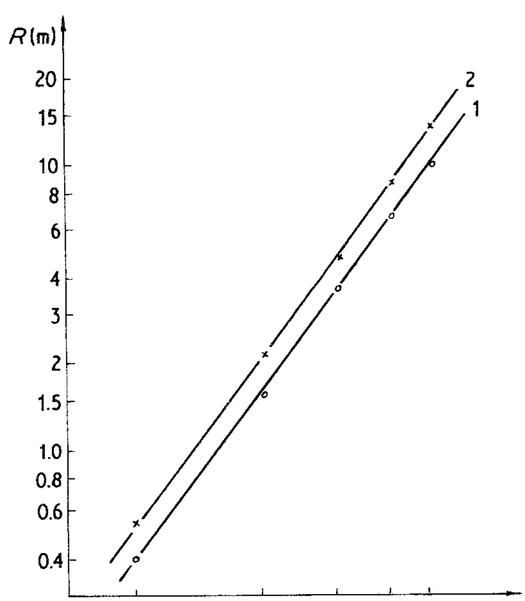


Figure 7 Dependence $R(h)$ for quartz monocrystal wafers: 1, before annealing; 2, after annealing.

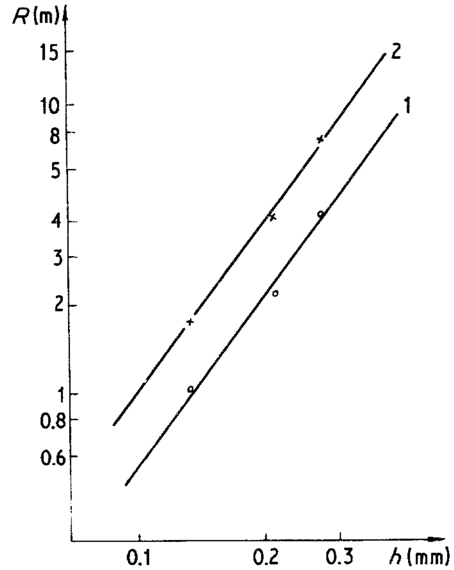


Figure 8 Dependence $R(h)$ for optical glass SF_{14}; 1, before annealing; 2, after annealing.

balanced by the surface tension of the opposite side torn during grinding. This tearing of the ground face increases the intermolecular distances and therefore the surface tension forces on this face cease to act or decrease to a great degree and are unable to balance the surface tension forces on the polished side. As a result, the wafer bends until the forces of surface tension are balanced by the additional inner elastic forces which have appeared in accordance with Hooke's law.

The surface energy of the polished side, U_p, is proportional to the wafer's area, S:

$$U_p = \sigma S, \qquad (2)$$

where σ is the coefficient of surface tension. When the wafer bends the change of energy, ΔU_p, is proportional to the change of the wafer's area, ΔS,

$$\Delta U_p = \sigma \Delta S. \qquad (3)$$

The coefficient σ can be obtained from the conditions for the minimum of full energy of the wafer.

The wafer's full energy can be represented as a sum of the energy of elastic deformation of the wafer, U_e, and the energy of the polished side, U_p. The energy, U_p, can be represented as a sum of the energy, $U_{p,const}$, of the unbent state and the change of energy, ΔU_p, caused by the bending and change of the area with ΔS, determined from Equation 3:

$$U_p = U_{p,const} + \Delta U_p. \qquad (4)$$

The full energy of the wafer will be:

$$U = U_{\mathrm{p,const}} + \Delta U_{\mathrm{p}} + U_{\mathrm{e}}. \qquad (5)$$

The energy of elastic deformation, U_e, of a spherically bent round wafer with a constant radius of curvature, R, can be easily obtained, using data from the theory of elasticity:

$$U_{\mathrm{e}} = \frac{E}{1-\mu}\frac{S_{\mathrm{m}}h^3}{12}x^2, \qquad (6)$$

where x is the wafer's curvature, E is the Young's modulus, μ is the Poisson's coefficient, and $S_m = \pi r^2$ is the area of the neutral middle plane of the wafer.

From the condition for the minimum of full energy, U, determined from Equation 5, differentiating with respect to x, we obtain:

$$\frac{\mathrm{d}U}{\mathrm{d}x} = 0 = \sigma\frac{\mathrm{d}\Delta S}{\mathrm{d}x} + \frac{E}{1-\mu}\frac{S_{\mathrm{m}}h^3}{6}x. \qquad (7)$$

It is evident from simple geometrical considerations that

$$\frac{\Delta S}{S_{\mathrm{m}}} = -\frac{h}{R} = -hx$$

or

$$\Delta S = -S_{\mathrm{m}}hx. \qquad (8)$$

By substituting Equation 8 into Equation 7 we obtain:

$$\frac{\mathrm{d}U}{\mathrm{d}x} = 0 = -\sigma S_{\mathrm{m}}h + \frac{E}{1-\mu}\frac{S_{\mathrm{m}}h^3}{6}x,$$

from where it follows that:

$$\sigma = \frac{1}{\sigma}\frac{E}{1-\mu}h^2x = \frac{1}{6}\frac{E}{1-\mu}\frac{h^2}{R}. \qquad (9)$$

From Equation 9 we obtain the dependence of R on the thickness of the wafers:

$$R = \frac{1}{6\sigma}\frac{E}{1-\mu}h^2$$

or

$$R = ah^2 \qquad (10)$$

where

$$a = \frac{1}{6\sigma}\frac{E}{1-\mu}. \qquad (11)$$

This derivation confirms the experimentally obtained law (Equation 1) for the dependence $R(h)$ and gives the physical sense of the constant a, depending solely on the characteristics of the material, E and μ, and the surface state. On the other hand, Equation 11 leads to:

$$\sigma = \frac{1}{6a}\frac{E}{1-\mu}. \qquad (12)$$

As can be seen from Equation 12, the surface tension, σ, defined above depends solely on the characteristics of the material E, μ and a, but it does not depend on the geometrical size of the investigated samples and is therefore a material constant. It can be easily determined by measuring the radius of curvature of spherically bent thin wafers cut out of different optical glasses and quartz or sapphire crystals (0 0 0 1), using Equation 12. The values thus obtained, together with the values of other material constants for the optical glasses, quartz crystals and metals we investigated are given in Table III.

We consider that the surface tension in solids defined by applying the Twyman effect is a contribution to the extremely important problem of surface tension and surface energy in solids. Most of the experimental and theoretical methods for defining the surface energy and surface tension in solids that exist to date are still unsatisfactory. Most of the experimental methods are indirect and involve destruction of the solid. Such are the methods of cleaving of the crystal, of measuring the heat of dissolution by dispersion, of subsiding oscillations and so on. The "zero creep method" is not associated with destruction, but with it σ is defined under high temperature, close to the melt-

TABLE III Values of Young's modulus, Poisson's coefficient and surface tension, σ, determined by the Twyman effect for optical glasses, quartz crystal and metals

Material	E (10^{-10} N m^{-2})	μ	σ (10^{-2} N m^{-1})
Quartz glass	5.9	0.33	2.10
BK_7	8.1	0.209	2.94
BaK_2	7.0	0.232	2.87
SF_4	5.4	0.244	2.77
Quartz crystal (0 0 0 1)	7.3	0.135	3.35
Copper	11.0	0.325	22.75
Silver steel	21.0	0.260	57.77

ing point, when the properties of the solid change considerably. These methods are so imperfect that they give values for σ sometimes differing by several orders of magnitude. With the theoretical methods the different authors try to calculate σ on the basis of various suppositions. The values for the surface energy of a given face of the crystal for one and the same substance are as numerous as the authors who have calculated them. There is still no general theory on these problems and we do not know which of the calculated values of σ are nearer to the real ones.

4. Applications of the Twyman effect

4.1. Application of the Twyman effect as a new method of studying anisotropy of crystals

The new method of studying anisotropy of crystals, based on the Twyman effect is, in appearance, similar to the classical method of heat conduction of a given crystal face [14], although it essentially differs from it.

For the anisotropic faces of quartz or sapphire monocrystals it is easy to see that the ratio of the radii of curvature, R'/R'', along the two axes of the elliptical rings is equal to the square of the ratio of their corresponding lengths, d_1/d_2:

$$\frac{R'}{R''} = \left(\frac{d_1}{d_2}\right)^2. \tag{13}$$

Consequently, according to our light-interference method, we can determine the degree of anisotropy which is evident from the ellipsoid bending, through one or the other ratio d_1/d_2 or R'/R''.

Table IV gives the ratios d_1/d_2 and R'/R'' at two thicknesses $h = 0.1$ and 0.2 mm for wafers of a quartz monocrystal cut out perpendicular to the X-axis, measured for several samples of equal thickness, while Table V gives analogous data for wafers of the second rhombohedral faces.

As is seen from the tables, the ratio d_1/d_2 (R'/R'' respectively) is the same for each face of the quartz monocrystal and it could serve to judge the degree of anisotropy.

TABLE IV A wafer of a quartz monocrystal cut out perpendicular to X

No.	$h = 0.1$ mm		$h = 0.2$ mm	
	d_1/d_2	R'/R''	d_1/d_2	R'/R''
1	1.16	1.34	1.16	1.33
2	1.18	1.38	1.15	1.37
3	1.19	1.39	1.18	1.39
av.	1.18	1.37	1.165	1.36

TABLE V A wafer of a quartz monocrystal cut out of the second rhombohedral faces

No.	$h = 0.1$ mm		$h = 0.2$ mm	
	d_1/d_2	R'/R''	d_1/d_2	R'/R''
1	1.05	1.11	1.06	1.11
2	1.07	1.15	1.05	1.12
3	1.03	1.10	1.05	1.12
4	1.07	1.11	1.06	1.11
av.	1.06	1.12	1.055	1.115

The data presented above demonstrate that the light-interference method of studying the anisotropy of crystals is simple and convenient. It is of interest probably as a preliminary method in studying the structure of crystals.

4.2. Application of the Twyman effect as a new method of preparing concave spherical mirrors for measuring equipment

According to the methods existing so far, concave spherical glass mirrors have usually been prepared by gradual grinding of suitably cut out samples of glass platelets with abrasive powder on spherically convex metal supports, usually made out of brass with a definite radius of curvature until the glass platelets acquire the same curvature. They are then polished on the same support with polishing powder and metallized to obtain better reflection of light. It is difficult to prepare very light mirrors of small sizes for sensitive equipment by this method.

The new method we suggest of preparing concave spherical mirrors of very small sizes for sensitive measuring equipment with a light-ray such as galvanometers, electrometers, voltmeters, oscillographs, etc., is based on the Twyman effect [15]. Using the technology we suggest [7], a whole series of spherically concave glass wafers can be produced. Their radii for a given kind of glass can be determined from Fig. 2 and Equation 1 and depend only on their thickness, h. In order to use these wafers as mirrors in measuring equipment, they are metallized by evaporating aluminium or silver in vacuum.

The advantage of this method is not only its technological simplicity, but also the possibility of producing thin and light concave spherical mirrors of miniature sizes – with a thickness of 0.5 to

0.1 mm and an area of 3 to 4 mm^2 for very sensitive measuring apparatus using a light ray. The radius of these mirrors changes in a broad range 20 to 0.4 m and it can be reduced even further by thinning the platelets to a respective thickness, $h < 0.1$ mm, defined by Fig. 2.

This method makes possible the production of spherical mirrors also of a greater size: with a diameter of 50 to 100 mm and considerably great radii of curvature, for use in small telescopes.

4.3. Application of the Twyman effect as a method of preparing a new type of spherical crystalline diffraction lattice for X-rays

There are several X-ray diffraction methods using a crystal lattice: those with a flat crystal (Bragg W.L., Bragg W.H. a. Soller), the methods of vertical focusing (Hamos, Kunzl) and of horizontal focusing (Johann, Johansson, Cauchois, Du Mond) [16]. The methods of vertical and horizontal focusing make use of single crystal wafers bent mechanically along the surface of a circular cylinder.

Our investigations of the Twyman effect of quartz and sapphire monocrystals enabled us to prepare a new type of spherical crystalline diffraction lattice for X-rays [17]. The Twyman effect, as already seen, gives an opportunity of obtaining spherical bending of a wafer, cut out from the (0001) plane of a quartz or sapphire monocrystal, as a result of its processing by the above technology.

The radius of curvature, as can be seen from Fig. 2, can vary within wide limits (from 0.5 to 20 m), depending solely on the thickness of the wafer. This means that the proposed method permits preparation of spherical crystalline wafers for X-ray spectroscopy with a pre-determined radius by thinning them, according to the technology described, to a respective thickness as determined by the plot in Fig. 2 (lines 5 and 6).

The spherical crystalline diffraction lattice obtained in this way may be expected to possess considerable advantages over the crystalline lattices mechanically bent by the above methods, i.e. greater intensity of the spectral lines at the same or greater resolution. Preliminary experiments have been made along this line.

5. Conclusions

The detailed investigations of the Twyman effect can be further carried out along a number of prospective lines:

1. investigation by this method of various solids (semiconductors, dielectrics, metals, etc);
2. investigation of the influence of X-rays, nuclear radiation, ion implantation, etc., on this phenomenon;
3. application of this effect as a method of investigating the surface tension or surface energy of two boundary phases (of a solid and gas, liquid or another solid phase, etc.);
4. we assume that the results obtained can be applied in integral optics, microelectronics, optoelectronics, etc.

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